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Slow-Wave Coplanar Waveguide on Periodically Doped Semiconductor Substrate

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Abstract —**A metal–insulator–semiconductor (MIS) coplanar waveguide with periodically doped substrate is described. An efficient numerical method is introduced in order to obtain the propagation constants and the characteristic impedances of the constituent sections of each period. Using the results, the characteristic of the periodic MIS coplanar waveguide is analyzed by Floquet's theorem. The theoretical study shows reduction of attenuation and enhancement of the slow-wave factor at certain frequencies, compared to the uniform MIS coplanar waveguide. This structure is experimentally simulated and shows good agreement theory.**

I. Introduction

RECENT STUDIES OF planar transmission lines on semiconductor substrates show the existence of slow-wave propagation [1], [2]. These transmission lines have either metal–insulator–semiconductor (MIS) configurations or Schottky-contacts on semiconductor substrates. The propagation speed depends greatly on the thickness of the insulating layer under the metal. Therefore, the bias dependence of the Schottky-contact depletion layer thickness can be advantageously used to construct variable phase shifters [3]. For applications in monolithic microwave integrated circuits, planar structures are preferred, of which the coplanar waveguide seems to be the most suitable structure because it is possible to connect series and parallel components. Therefore, we shall limit our discussion to this structure. To obtain slow-wave propagation over a wide frequency range, it is necessary to raise the conductivity of the semiconductor substrate to a large value, typically $10^4 \sim 10^5$ mho/m. This, however, causes high attenuation at high frequencies and makes practical applications difficult.

Several periodic structures have been proposed to reduce this attenuation [4], [5]. These are ideally lossless waveguides, and the slow-wave propagation is obtained because of the periodicity. The present structure alternatively introduces lossless sections periodically to the MIS or Schottky coplanar waveguide. By doing this, the important phase-shift property of the Schottky-contact coplanar waveguide is still preserved. In addition to reducing the attenuation, the periodicity of the present structure exhibits an inherent slow-wave nature. Therefore, an improvement of the propagation characteristic is expected.

The theoretical study in this paper shows a possibility to extend the frequency range of the slow-wave propagation. Also, an experimental model was built and tested to verify this theoretical calculation.

II. Theory

The basic theoretical treatment of the MIS periodic coplanar waveguide consists of using Floquet's theorem for periodic transmission lines. Fig. 1(a) shows a schematic view of the structure. A coplanar waveguide is placed, via an insulating layer of thickness d_1, on the semi-insulating

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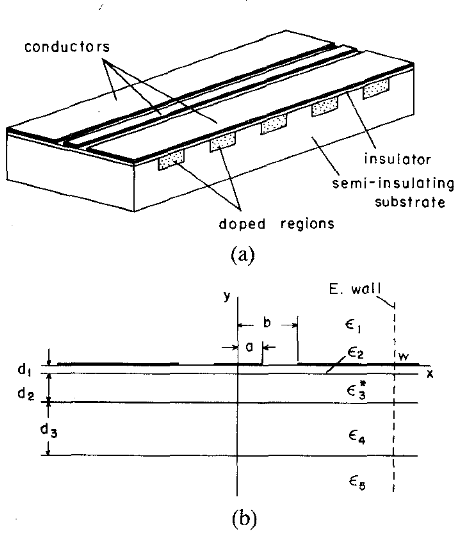

Fig. 1. Schematic view of the structure. (a) MIS periodic coplanar waveguide. (b) Cross-sectional view of each section. Lossy section: $\epsilon_3^* = \epsilon_3 + \sigma/j\omega$. Lossless section: $\epsilon_3^* = \epsilon_3 = \epsilon_4$.

semiconductor substrate, which is periodically doped to form highly conductive regions. To utilize Floquet's theorem, we first need to calculate the propagation constants and characteristic impedances for the constituent sections of each period. These are computed by a hybrid-mode analysis technique, which was proposed by Yamashita and Atsuki [6]. The metalization is assumed to be infinitesimally thin, and for the calculation of the lossy section, the cross section is divided into five regions (Fig. 1(b)). Each region is characterized by a different dielectric constant. The conductivity σ of the doped region is assumed to be constant and incorporated into a complex dielectric constant ϵ_3^*. Because of the symmetry with respect to the y-axis, we need consider only the right half of the structure. A vertical electric wall needs to be placed at the far right ($x = w, w \gg b$) to generate a discrete Fourier series solution. The solution in each region is expressed in terms of two scalar potentials, i.e., magnetic and electric potentials with respect to the y-direction. After considering all the boundary conditions, the following integral equation is finally formulated:

$$\int_s E_x(x_0) K_1(x|x_0)\, dx_0 + \int_c J_z(x_0) K_2(x|x_0)\, dx_0 = 0, \qquad 0 < x < w. \quad (1)$$

$E_x(x)$ represents the x-component of the electric field in the slot region, and $J_z(x)$ the z-component of the current density on the conductors. Since these two unknown functions are defined in different regions, we can treat them as one unknown function. Therefore, only one integral equation is needed. The integration of the first term of (1) is over the slot region ($a < x < b$), and the second term is over the conductors ($0 < x < a, b < x < w$). The expressions of the kernels $K_1(x|x_0)$ and $K_2(x|x_0)$ of the integral equation (1) are given in the Appendix. Equation (1) is solved by nonuniform discretization (N_p matching points), which yields the complex propagation constant γ_a of the

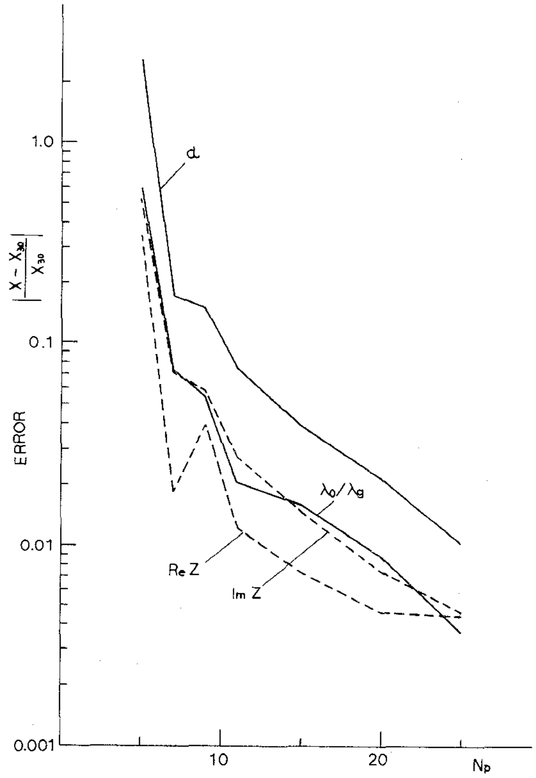

Fig. 2. Convergence of the solutions. $a = 0.05$ mm, $b = 0.5$ mm, $d_1 = 1.0$ μm, $d_2 = 3.0$ μm, $d_3 = 1.0$ mm, $\epsilon_2 = 8.5\epsilon_0$, $\epsilon_3 = \epsilon_4 = 13\epsilon_0$, $\sigma = 10^4$ mho/m, frequency = 0.1 GHz. ($N_p = 30$) $\lambda_0/\lambda_g = 22.321$, $\alpha = 0.01759$ dB/mm, $Z_c = 9.733 + j0.3901$ Ω.

lossy section. Using the solution of the integral equation $E_x(x)$ and $J_z(x)$, the characteristic impedance of this section is easily calculated as follows:

$$Z_a = \frac{\int_a^b E_x(x)\, dx}{2\int_0^a J_z(x)\, dx}. \quad (2)$$

The lossless section can also be analyzed using this same method by setting $\sigma = 0$, which yields γ_b and Z_b.

The overall propagation constant of the MIS periodic coplanar waveguide γ is then approximately calculated by applying the Floquet's theorem:

$$\cos(\gamma l) = \cos(\gamma_a l_a)\cos(\gamma_b l_b) - \frac{1}{2}\left[\frac{Z_a}{Z_b} + \frac{Z_b}{Z_a}\right]\sin(\gamma_a l_a)\sin(\gamma_b l_b)$$

$$l = l_a + l_b : l_a, l_b \cdots \text{length of each section.} \quad (3)$$

The use of this equation neglects the higher order effects of the geometric junction discontinuities. The slow-wave factor and attenuation constant are obtained from the real and imaginary parts of γ, respectively.

III. Numerical Results

Numerical solution of the integral equation is very efficient and successful. An example of the convergence of the solutions is shown in Fig. 2, which is calculated for a

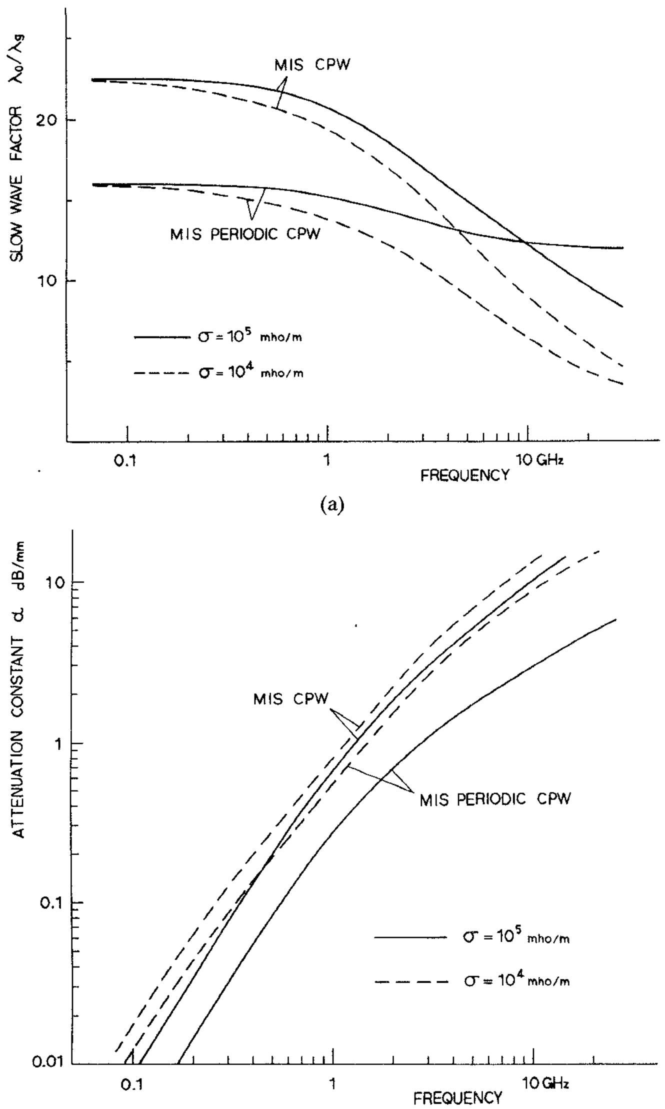

Fig. 3. Comparison of MIS periodic coplanar waveguide with uniform MIS coplanar waveguide. (a) Slow-wave factor versus frequency. (b) attenuation constant versus frequency. $a = 0.05$ mm, $b = 0.5$ mm, $d_1 = 1.0$ μm, $d_2 = 3.0$ μm, $d_3 = 1.0$ mm, $\epsilon_2 = 8.5\epsilon_0$, $\epsilon_3 = \epsilon_4 = 13\epsilon_0$, $l_a = l_b = 0.1$ mm.

typical uniform MIS coplanar waveguide. The figure shows errors relative to the values at $N_p = 30$. Convergence becomes even faster for the waveguide having a thicker insulator or narrower slots. The computed results were also compared with other methods, such as the spectral-domain analysis and the mode-matching method, and were proven to be accurate [7].

The computed slow-wave factors and attenuation constants for the MIS periodic coplanar waveguide are presented in Fig. 3(a) and (b), respectively. Two typical cases are shown in the figures. For the case $\sigma = 10^5$ mho/m (solid curves), the extension of the frequency range of the slow-wave propagation is observed. Namely, the slow-wave factor of the periodic structure becomes greater than that of the uniform MIS coplanar waveguide at frequencies higher than 10 GHz. This crossover of the slow-wave factor occurs for conductivities larger than a certain critical value. For instance, no crossover point exists for $\sigma = 10^4$ mho/m (dotted curves). This difference can be explained by the behavior of the doped layer. A highly conductive doped layer tends to act like a conductor at high frequencies [1]. Therefore, the effect of periodicity becomes strong and provides slow-wave propagation in this frequency range. In fact, if the doped regions of the MIS periodic coplanar waveguide are replaced by conductor strips, the resulting slow-wave factor follows nearly the same curve at high frequencies. On the other hand, a doped layer with low conductivity acts as a dielectric layer, and the periodicity becomes weak at high frequencies. Hence, in this case, the periodic structure does not provide the enhancement to the slow-wave behavior. The reduction of attenuation is also more pronounced for the case $\sigma = 10^5$ mho/m compared with the other case (Fig. 3(b)). Therefore, the advantage of introducing the periodic sections is obtained when the conductivity of the doped regions is large.

IV. Experiments

A model of the MIS periodic coplanar waveguide was fabricated and tested in the frequency range of 40 MHz–1 GHz. Instead of a doped semiconductor substrate, graphite powder was used as a resistive material. The graphite powder was sandwiched by two adhesive plastic sheets, and placed periodically on the coplanar waveguide etched on a circuit board. A model of the uniform MIS coplanar waveguide is also fabricated using the same materials. The dimensions and other parameters of these two models are as follows.

Common Parameters

Length of line = 212 mm

$$a = 1.0 \text{ mm}, b = 5.0 \text{ mm}$$

$$\epsilon_1 = \epsilon_2 = \epsilon_4 = 2.5\epsilon_0 \ (\epsilon_2 = \epsilon_0 \text{ for lossless section})$$

$$\epsilon_3 = \epsilon_5 = \epsilon_0, \sigma \simeq 20 \text{ mho/m (estimate)}.$$

MIS Periodic Coplanar Waveguide

$$d_1 = 0.13 \text{ mm}, d_2 = 0.02 \text{ mm}, d_3 = 3.0 \text{ mm}$$

$$l_a = l_b = 8.0 \text{ mm (12 periods)}.$$

Uniform MIS Coplanar Waveguide

$$d_1 = 0.18 \text{ mm}, d_2 = 0.02 \text{ mm}, d_3 = 3.0 \text{ mm}.$$

The input impedances of the lines with open and short ends were measured using an admittance bridge, and the results were converted into the slow-wave factors and attenuation constants. The conductivity of the graphite layer was estimated from the measurement of the propagation constant of the simulated uniform MIS coplanar waveguide. The dielectric constant of the graphite layer was assumed to be unity. The experimental results are shown in Fig. 4. These are in reasonably good agreement with the theoretical curves in spite of the nonuniformity in the thickness and densities of the graphite layers. The crossover of the slow-wave factor does not occur in this case since the conductivity of the graphite powder is too

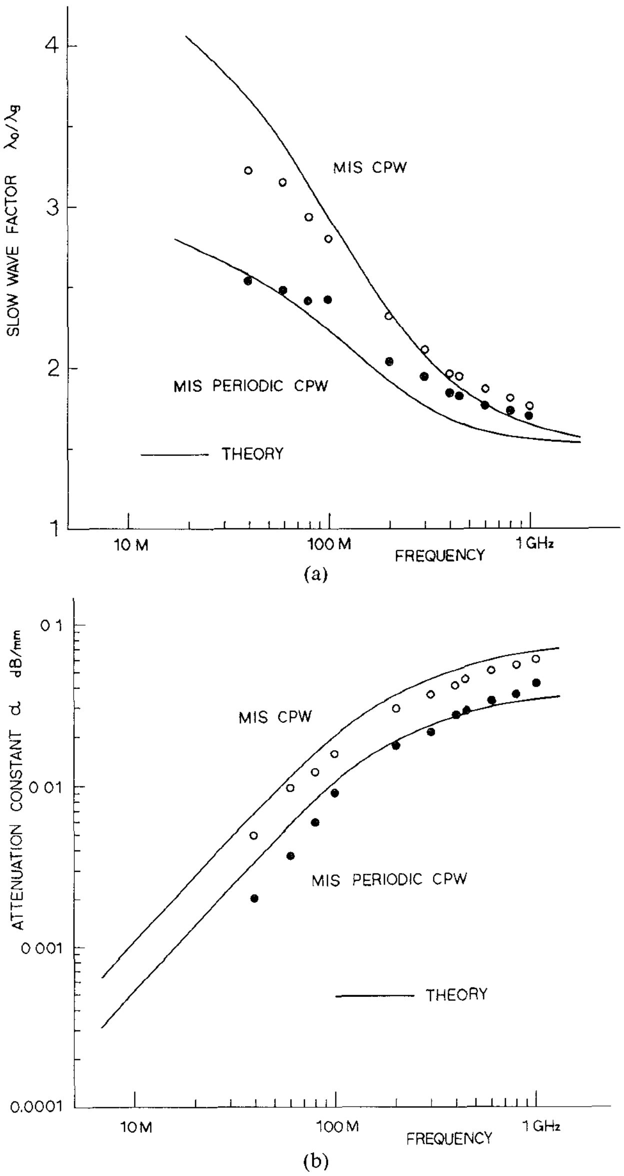

Fig. 4. Experimental results. (a) Slow-wave factor versus frequency. (b) Attenuation constant versus frequency.

small. However, this experiment verifies the theoretical calculation, and the extension of the slow-wave propagation range is expected if appropriate conductivity is chosen.

V. Conclusions

MIS periodic coplanar waveguide was analyzed and the theory tested by experiment. The result shows the reduction of attenuation and the extension of the frequency range of the slow-wave propagation.

The present structure can also be used as a variable phase shifter if MIS sections are replaced with Schottky-contact coplanar waveguides. In this case, reduced attenuation is also expected and, therefore, operation of such a voltage-controlled phase shifter at higher frequency may be possible.

Appendix
Derivation of the Integral Equation (1)

The magnetic and electric potentials in the air region ($y > 0$, dielectric constant ϵ_1) and the insulator region ($-d_1 < y < 0, \epsilon_2$) are

$$\psi_1 = \sum_n a_n \cos k_{xn} x e^{-k_{y1n} y}$$

$$\phi_1 = \sum_n b_n \sin k_{xn} x e^{-k_{y1n} y}, \qquad y > 0 \qquad \text{(A1)}$$

$$\psi_2 = -\sum_n \frac{\epsilon_2 k_{y1n}}{\epsilon_1 k_{y2n}} a_n \cos k_{xn} x (\sin k_{y2n} y + P_{cn} \cos k_{y2n} y)$$

$$\phi_2 = \sum_n b_n \sin k_{xn} x (P_{dn} \sin k_{y2n} y + \cos k_{y2n} y), \qquad -d_1 < y < 0 \qquad \text{(A2)}$$

where the summations over n range from one to infinity, a_n and b_n are unknown constants, and

$$k_{xn} = (n - 1/2)\pi / w$$

$$k_{y1n}^2 = \gamma^2 + k_{xn}^2 - \omega^2 \epsilon_1 \mu_0$$

$$k_{y2n}^2 = \omega^2 \epsilon_2 \mu_0 - \gamma^2 - k_{xn}^2.$$

P_{cn} and P_{dn} in (A2) are constants and determined by matching boundary conditions at $y = -d_1$, $-(d_1 + d_2)$, $-(d_1 + d_2 + d_3)$. The expressions (A1) and (A2) already satisfy the continuity condition of tangential electric field components (E_x and E_z) at $y = 0$. Now we require E_x and E_z to be zero on the conductors ($0 < x < a$, $b < x < w$) and the tangential magnetic field components (H_x and H_z) to be continuous in the slot region ($a < x < b$). Using two unknown functions, E_x in the slot region and J_z (current density) on the conductors, we can obtain the integral equation (1). The kernals are given by

$$K_1(x|x_0) = \begin{cases} \sum_n A_n \sin k_{xn} x_0 \cos k_{xn} x, & 0 < x < a, \\ & b < x < w \\ \sum_n C_n \sin k_{xn} x_0 \sin k_{xn} x, & a < x < b \end{cases}$$

$$K_2(x|x_0) = \begin{cases} \sum_n B_n \cos k_{xn} x_0 \cos k_{xn} x, & 0 < x < a, \\ & b < x < w \\ \sum_n A_n \cos k_{xn} x_0 \sin k_{xn} x, & a < x < b \end{cases}$$

where

$$A_n = \frac{1}{V_n} \left[\frac{1}{\omega^2 \epsilon_1 \mu_0} k_{y1n}^2 T_n + S_n \right] k_{xn} \gamma$$

$$B_n = \frac{1}{V_n} (\gamma^2 + k_{xn}^2) \frac{k_{y1n}}{\omega \epsilon_1}$$

$$C_n = \frac{1}{V_n} (\gamma^2 + k_{xn}^2) \frac{k_{y1n}}{\omega \mu_0} S_n T_n$$

$$V_n = \gamma^2 S_n - \frac{k_{y1n} k_{xn}}{\omega^2 \epsilon_1 \mu_0} T_n$$

$$S_n = 1 + P_{cn} \frac{\epsilon_2 k_{y1n}}{\epsilon_1 k_{y2n}}$$

$$T_n = 1 + P_{dn} \frac{k_{y2n}}{k_{y1n}}.$$

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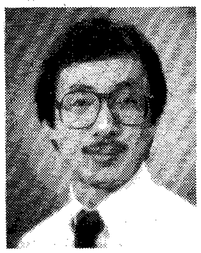

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Design of an Overlay Directional Coupler by a Full-Wave Analysis

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***Abstract*— A full-wave analysis based on the spectral-domain method is applied to coupled overlay microstrips, coupled inverted microstrips, and coupled microstrips. Exclusive numerical data including frequency characteristics are included. A 10-dB overlay coupler was built according to the design theory, and experimental results are reported.**

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I. Introduction

IT IS KNOWN THAT when the even- and odd-mode propagation constants are identical, the isolation of a directional coupler is theoretically infinite. However, in an inhomogeneous structure such as microstrip, this condition is not always satisfied. A dielectric overlay is one way to improve the isolation of a microstrip coupler, by which the difference in even- and odd-phase velocities can be greatly reduced or even equalized [1]–[3]. To date, most of the

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